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COMMON REFLECTION POINT DATA-STACKING TECHNIQUE FOR CONVERTED WAVES[1]

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ABSTRACT

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For converted waves stacking requires a true common reflection point gather which, in this case, is also a common conversion point (CCP) gather. We consider converted waves of the PS- and SP-type in a stack of horizontal layers.

The coordinates of the conversion points for waves of PS- or SP-type, respectively, in a single homogeneous layer are calculated as a function of the offset, the reflector depth and the velocity ratio v_p/v_s. Knowledge of the conversion points enables us to gather the seismic traces in a common conversion point (CCP) record. Numerical tests show that the CCP coordinates in a multilayered medium can be approximated by the equations given for a single layer. In practical applications, an *a priori* estimate of v_p/v_s is required to obtain the CCP for a given reflector depth.

A series expansion for the traveltime of converted waves as a function of the offset is presented. Numerical examples have been calculated for several truncations. For small offsets, a hyperbolic approximation can be used. For this, the rms velocity of converted waves is defined. A Dix-type formula, relating the product of the interval velocities of compressional and shear waves to the rms velocity of the converted waves, is presented.

INTRODUCTION

Stacking techniques for common reflection point data are commonly used in reflection seismology to attenuate multiples and random noise and to estimate the subsurface velocity distribution. The application of this technique to converted waves of PS- or SP-type is not as simple as for PP- or SS-reflections, which essentially have symmetrical wave paths.

The path of the converted wave is asymmetrical, even for the simple, horizontally layered medium which is considered in this paper. Multiple coverage cannot therefore be achieved by a common midpoint (CMP) gather but requires a true common reflection point sorting which, in this case, yields a common conversion point (CCP) gather.

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We consider the first steps towards common reflection point stacking and velocity analysis for converted waves. These steps are: (1) common reflection or common conversion point sorting of data, and (2) traveltimes and rms velocities of converted waves.

Sorting of Traces with a Common Conversion Point

Several authors (e.g. Behle and Dohr 1985) have discussed the problems associated with the gathering of converted wave data of common conversion points. The sorting depends on the depth of the reflector and on the ratio of compressional and shear-wave velocity v_p/v_s. For complex models the conversion point of the wave can probably be established only by iterative methods, e.g. ray tracing. For a single horizontal homogeneous layer, Fromm, Krey and Wiest (1985) derived the relation

$$x_p = \frac{x}{1 + (v_s/v_p)} \tag{1}$$

as a first-order approximation for the horizontal distance x_p of the conversion point from the source point. Chung and Corrigan (1985) used this relation for a horizontally layered medium to gather mode-converted shear waves in synthetic data. The same relation, expressed in a slightly different form, was also used to stack field data by Frasier and Winterstein (1986), who derived it from equations given by Nefedkina (1980).

To obtain a focusing effect for a single reflector at arbitrary depth, the variation of the conversion point coordinate x_p with depth is significant (cf. Behle and Dohr 1985) and must be considered. The conversion point of a single horizontal reflector

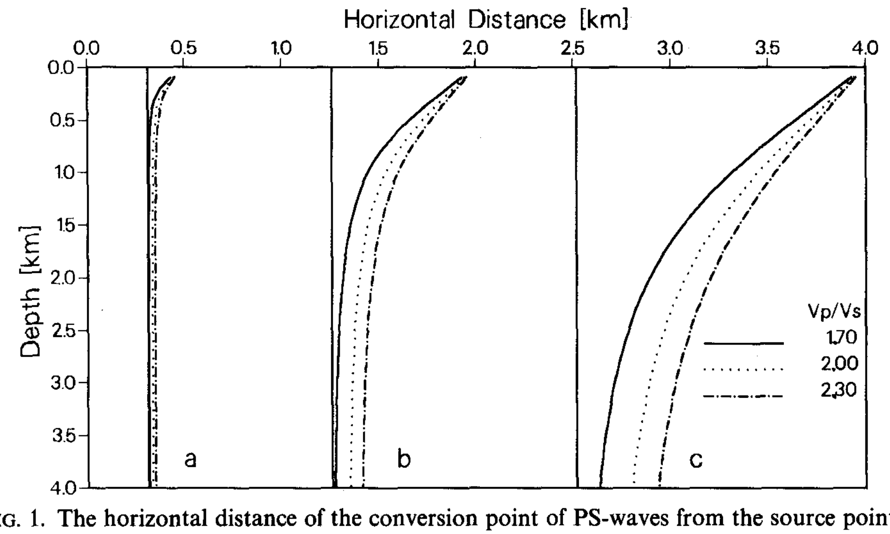

FIG. 1. The horizontal distance of the conversion point of PS-waves from the source point as a function of the depth of a single reflector for several ratios v_p/v_s and an offset (a) 0.5 km, (b) 2.0 km, and (c) 4.0 km. The straight lines have been calculated from (1) with $v_p/v_s = 1.7$.

can be calculated as a function of the reflector depth z and the velocity ratio v_s/v_p for a given offset x. A detailed description is given in Appendix A.

In Fig. 1 the conversion points of PS-waves calculated from (A15) are plotted as a function of reflector depth for three selected offsets (0.5, 2.0 and 4.0 km respectively) and three different values of the ratio v_p/v_s. The range between 1.7 and 2.3 may be considered as representative in practice excluding the extremely high values found in shallow layers. For example, when the depth of the reflector is 2.3 km and the ratio v_p/v_s is 2.0, the horizontal distance of the conversion point from the source point is 3.0 km when the offset is 4.0 km.

The horizontal distances x_p of the conversion points from the shot point as calculated from (1) for a velocity ratio of 1.7 are plotted for comparison. As these values are independent of depth (cf. (1)) the resulting curves in Fig. 1 are vertical straight lines, which represent the small angle of incidence approximation for the horizontal coordinate of the conversion point. For shallow reflectors the approximation of the conversion point by (1) may introduce large errors whereas (A15) gives an accurate result for every depth.

An equivalent result can be obtained for SP-converted waves. Figure 2 shows the horizontal distance x_s of an SP-wave conversion point from the source point as a function of the reflector depth and three selected values of v_p/v_s when the offset is 0.5, 2.0 and 4.0 km, respectively. In this SP-case the asymptotes are given by

$$x_s = \frac{x}{1 + (v_p/v_s)}, \tag{2}$$

which is analogous to (1).

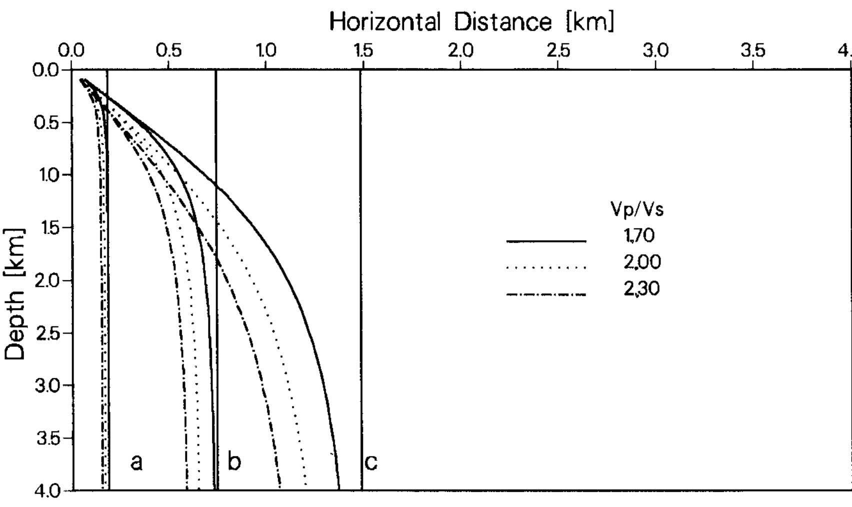

FIG. 2. The horizontal distance of the conversion point of SP-waves from the source point as a function of the depth of a single reflector for several ratios v_p/v_s and an offset (a) 0.5 km, (b) 2.0 km, and (c) 4.0 km. The straight lines have been calculated from (2) with $v_p/v_s = 1.7$.

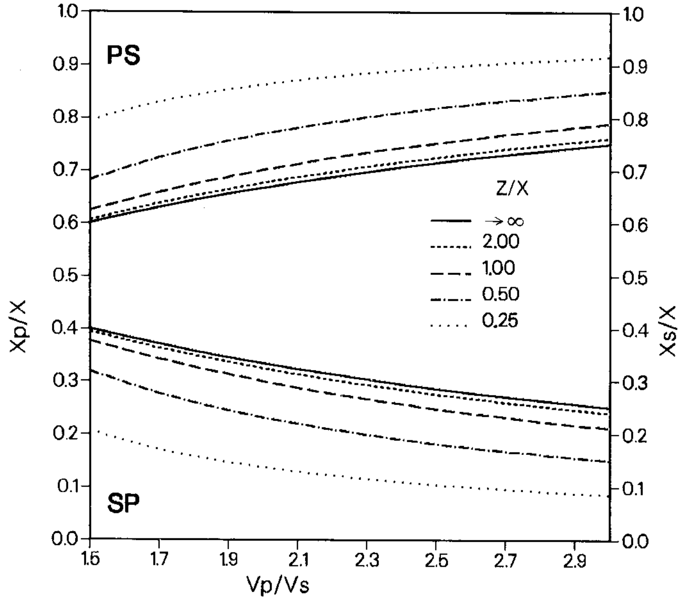

FIG. 3. The horizontal coordinates of the conversion points of PS- and SP-waves, normalized on the offset x_p/x and x_s/x, respectively, as a function of the ratio v_p/v_s for several ratios z/x.

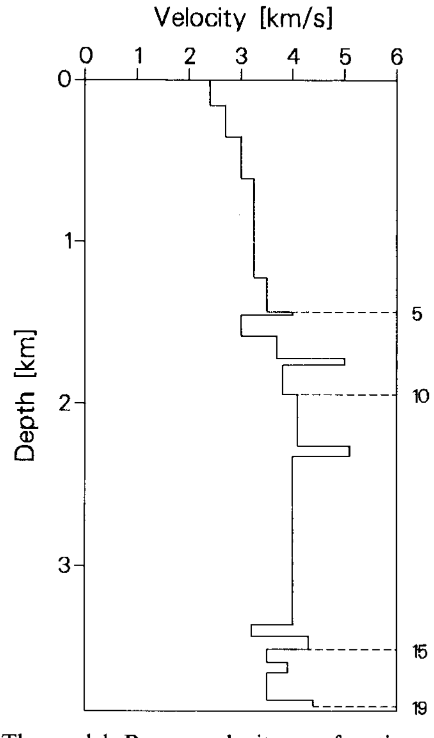

FIG. 4. The model: P-wave velocity as a function of depth.

The diagram in Fig. 3 shows the dependence of the horizontal coordinate of the conversion point on the velocity ratio v_p/v_s and the reflector depth, as obtained from (A15) and (A16) for PS- and SP-waves, respectively. Normalized values x_p/x, x_s/x and z/x have been used. To demonstrate the use of the diagram let us first assume that we want to estimate the change in x_p for a certain change in v_p/v_s at a fixed depth. If we have an offset of 1 km and a reflector at 1 km, i.e. $z/x = 1$, a change of v_p/v_s by 0.1 from 1.9 to 2.0 will cause a change of 0.013 in x_p/x so that the abscissa of the conversion point will change by 13 m. Conversely the diagram can also be used to quantify the change in the conversion point abscissa x_p/x for a change in reflector depth. Let us assume that we have calculated x_p of a conversion point at a reflector depth of 1 km and for an offset of 1 km (so that $z/x = 1$) assuming a v_p/v_s ratio of 2.1. If the reflector depth changes to 0.5 km, i.e. $z/x = 0.5$, then the curves in Fig. 3 indicate that x_p/x will increase by 0.07, i.e. x_p will increase by 70 m. If the change in x_p obtained from the diagram is less than one-half of the geophone group spacing, then it can be neglected. The coordinates of the conversion point have been calculated for one horizontal reflector so far. They can, however, also be used as an approximation for a stack of horizontal layers.

To show this, the true conversion points have been calculated by ray shooting and compared with the conversion points calculated from (A15) for an equivalent homogeneous layer of the same total thickness. The comparison has been made for several models which have been derived from the simplified acoustic log shown in Fig. 4. For example, the results of the comparison are shown in Fig. 5 for three

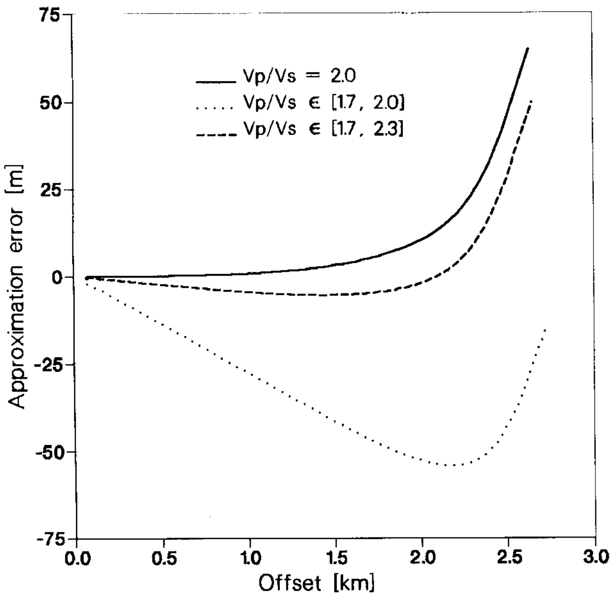

FIG. 5. The approximation error, i.e. the distance of the conversion point, calculated from (A15) with $v_p/v_s = 2.0$, from its position determined by ray shooting, as a function of the offset for three models with different ratios v_p/v_s in each of the ten layers of the log. (Note that the offset range is restricted by the critical angle of conversion $\beta^* = \arcsin(v_p/v_s)$.)

different models. In all these models the P-wave velocity distribution $v_p(z)$ is the same, whereas v_p/v_s has been varied. The models consist of the first ten layers of the log (Fig. 4). In the first model the ratio v_p/v_s is the same for all layers and is equal to 2.0. For the second and third models the ratios v_p/v_s are different in each layer and represent random numbers within the intervals 1.7–2.3 and 1.7–2.0, respectively. The equivalent homogeneous layer used for comparison with each of the three models is characterized by a velocity ratio $v_p/v_s = 2.0$.

Figure 5 shows the approximation error δx versus offset, where δx is the horizontal distance between the true conversion point coordinate $x_{\mathrm{p,\,true}}$ (calculated by ray shooting) and the straight-ray approximation for an equivalent homogeneous layer $x_{\mathrm{p,\,approx.}}$ as explained above:

$$\delta x = x_{\mathrm{p,\,true}} - x_{\mathrm{p,\,approx.}}. \tag{3}$$

For the three models considered, the maximum approximation error δx is 65 m for offsets up to 2.7 km. For deeper reflection zones the approximation errors are smaller for this model, even for slightly larger offsets.

These examples show that, if the depth of the reflector is not too small, the seismic traces can be gathered with respect to a common conversion point (CCP) using (A15) as an approximation for the conversion point in a multilayered medium. The assumption of the v_p/v_s ratio which has to be made for the CCP sorting can either be an *a priori* estimate or be derived from previous experience (shear-wave well logging or SS-wave survey) in or near the area of investigation.

Note that for a given offset the seismic trace may belong to different CCP families with respect to different horizons, i.e. for a given CCP different traces have to be gathered with respect to different horizons to achieve the optimum focusing effect. The CCP sorting of traces is considered to be superior to CDP sorting, particularly where the PS-reflection coefficient of the reflector under consideration shows considerable lateral changes.

Traveltimes and Velocities of Converted Waves

Taner and Koehler (1969) have shown that the traveltime of a wave which has been reflected from the nth interface of a horizontally layered medium can be expanded into a power series as follows

$$t_n^2(x) = c_1 + c_2 x^2 + c_3 x^4 + c_4 x^6 + \cdots . \tag{4}$$

The coefficients $c_i(n)$ have been derived for waves with a symmetrical wave path (PP, SS). Janle (1981) used this expansion to calculate the traveltime of a PS-converted wave for a single reflector.

We present an equivalent series expansion for a certain class of converted PS- and SP-waves in a horizontally layered medium. We consider only those waves for which we have one P- and one S-leg in each layer, regardless of whether they are up- or downgoing. For reflections of this type the squared traveltime is an even

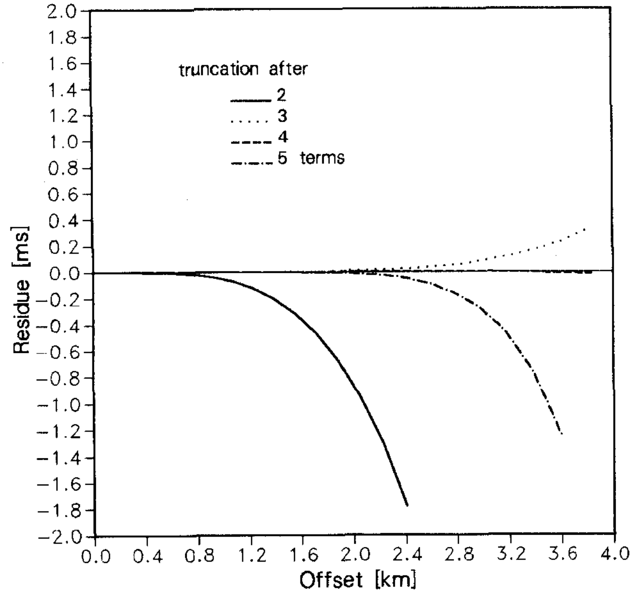

FIG. 6. The time residue, i.e. the difference between the exact traveltime and the approximated traveltime of PS-waves as a function of the offset.

function of x, as for PP-waves, and it can be shown that the coefficients have a similar form as for non-converted waves. Explicit formulae are given in Appendix B. Comparing the expansion coefficients for PP-waves with those for PS-waves we can formally define the rms velocity for a PS-reflection from the nth reflector by

$$\tilde{v}_{\mathrm{ps}_n} = \sqrt{\left[\frac{\sum_{k=1}^{n} (v_{\mathrm{p}_k} + v_{\mathrm{s}_k})h_k}{\sum_{k=1}^{n} (1/v_{\mathrm{p}_k} + 1/v_{\mathrm{s}_k})h_k}\right]}, \tag{5}$$

where v_{p_k} and v_{s_k} are the compressional and shear wave velocity of the kth layer respectively, and h_k is its thickness. For the bottom reflector (no. 19) of the model shown in Fig. 4 the traveltimes of PS-reflections have been approximated by truncating the power series after the second, third, fourth and fifth term respectively. These approximate values have been subtracted from the exact traveltimes calculated by ray shooting. The residues are shown in Fig. 6 as a function of the offset. For comparison the corresponding residues for PP-reflections are shown in Fig. 7.

The series for PS-reflection traveltimes does not converge as rapidly as the series for PP-reflections. Nevertheless, the second term truncation which represents a hyperbola can be used to give a satisfactory approximation for PS-moveout corrections in this case. This has also been confirmed by some tests on field data, results of which will be presented in a forthcoming paper.

To investigate the influence of the reflector depth on the hyperbolic approximation, the residuals of the truncation after the second term have been calculated for the fifth, tenth, fifteenth and (again) nineteenth reflector of the same model (Fig. 4). The corresponding reflectors are at depths of about 1.5, 2.0, 3.6 and 4.0 km

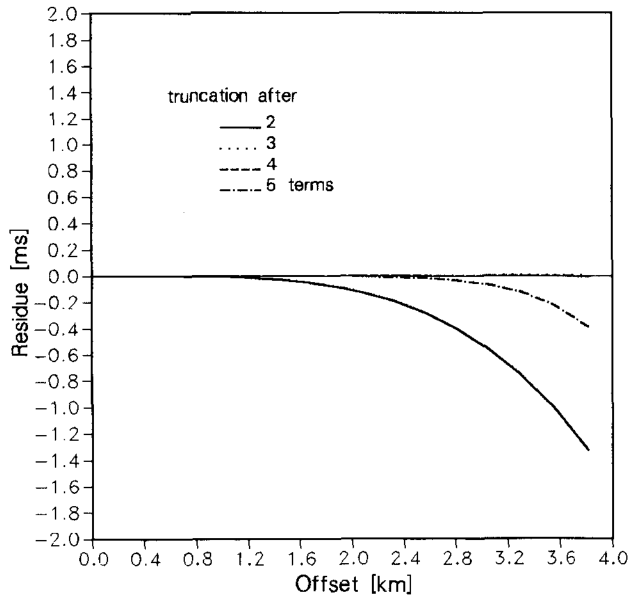

FIG. 7. As Fig. 6 but for PP-waves.

respectively. The resulting time residuals are shown in Fig. 8. With decreasing reflector depth, the offset range for which the hyperbolic approximation stays within a tolerable limit of 1 ms decreases down to 1 km for this model. These examples, as well as analytical calculations, show that in the near-offset range the hyperbolic approximation for the PS-traveltime curve is (under normal v_p/v_s conditions) as good as that for the PP-curve up to about 75% of the corresponding PP-offset.

If we assume the layers above the reflector under consideration to be laterally homogeneous, then the traveltimes on any trace of given offset are identical. Therefore the traveltime approximations derived above for a single shot can be used to calculate normal moveout corrections as a function of offset for CCP gathers as well. Thus standard processing techniques can be applied to a CCP gather within the offset range discussed above to produce a PS-reflection stack for the horizon on which the CCP gather has been focused. The depth range for which focusing can be achieved with sufficient accuracy can be estimated from (A15) in Appendix A or Fig. 3.

Also, a standard velocity analysis can be applied to the CCP gather to determine the PS-stacking velocity, which can be used to produce a stacked time section. This will of course depend on the *a priori* estimate of v_p/v_s used for the CCP sorting. To establish this dependence, additional quantitative analysis is required. As the CCP sorting also depends on the depth of the reflector, it may be necessary to re-gather the data in order to achieve CCP-focusing also at other desired depth ranges. The focusing quality obtained for a certain v_p/v_s ratio used for the sorting of traces can be assessed in the time window considered by comparing corresponding reflection patterns (if available) on the PS-time section with those on the PP- (or SS-) time

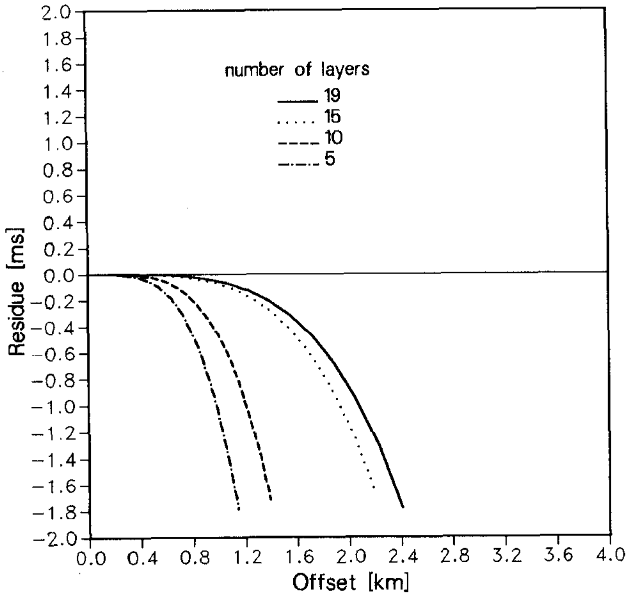

FIG. 8. As Fig. 6 but only two-term-truncation and for various numbers of layers.

section. In this way a complete PS-time section can be obtained by joining together the respective time windows for which satisfactory focusing has been achieved.

To minimize the effort of a PS-velocity analysis we can give a rough estimate for the PS-stacking velocity if the PP-stacking velocity function is available. Let us assume that the velocity ratio of compressional and shear waves v_p/v_s is the same for all layers:

$$\frac{v_{pk}}{v_{sk}} = \frac{v_p}{v_s} = \text{constant}. \tag{6}$$

We then obtain

$$\tilde{v}_{ps} = \tilde{v}_{pp}(v_s/v_p)^{1/2} = \sqrt{(\tilde{v}_{pp}\,\tilde{v}_{ss})}. \tag{7}$$

Thus, when using the rms velocity as an estimate of the stacking velocity, a velocity function for PS-reflections can be approximately derived from the P-wave stacking velocity analysis and some additional estimate of an average v_p/v_s ratio.

Normally interval velocities are determined by means of a Dix-type formula. Such a formula can also be derived for converted waves as presented in Appendix C. It should be noted, however, that for PS- and SP-waves this formula only allows the calculation of the product $v_p\,v_s$ of compressional and shear wave velocity. An independent P-wave (or S-wave) velocity analysis is required for the same time interval to yield the ratio v_p/v_s for that interval. The interval velocities can be determined only if interval compressional or shear wave velocities are available from the respective velocity analysis.

The interval velocity ratio v_p/v_s can be calculated directly from the interval traveltimes of the converted waves $\Delta t_n^{(ps)}$ and the interval traveltimes of the P-wave $\Delta t_n^{(pp)}$ or S-wave $\Delta t_n^{(ss)}$ from the corresponding time sections as follows:

$$\Delta t_n^{(ps)} = t_{0_n}^{(ps)} - t_{0_{n-1}}^{(ps)} = h_n\left(\frac{1}{v_{p_n}} + \frac{1}{v_{s_n}}\right),$$

and

$$\Delta t_n^{(ss)} = t_{0_n}^{(ss)} - t_{0_{n-1}}^{(ss)} = \frac{2h_n}{v_{s_n}},$$

or

$$\Delta t_n^{(pp)} = t_{0_n}^{(pp)} - t_{0_{n-1}}^{(pp)} = \frac{2h_n}{v_{p_n}},$$

respectively. Elimination of h_n yields

$$\frac{v_{p_n}}{v_{s_n}} = \frac{\Delta t_n^{(ss)}}{2\Delta t_n^{(ps)} - \Delta t_n^{(ss)}},$$

and

$$\frac{v_{p_n}}{v_{s_n}} = \frac{2\Delta t_n^{(ps)} - \Delta t_n^{(pp)}}{\Delta t_n^{(pp)}}.$$

This result is in agreement with a formula given by Garotta (1987) for average velocities. An essential requirement is the correct correlation of the events in the PP-section to those in the PS-sections. This can introduce difficulties, especially when the conversion properties of the interfaces differ strongly from their reflection properties. Additional problems can be caused, for example, by different tuning effects with respect to thin layers for different wave types, by effects of anisotropy or by insufficient shear wave static corrections. These problems have discussed, e.g. by McCormack, Dunbar and Sharp (1984) and Garotta (1985), concerning the correlation of PP- and SS-reflections. We believe much of this experience is valid for converted wave reflections as well.

Conclusion

It has been shown that the conversion point of a PS- or SP-type reflection in a horizontally layered medium can be determined approximately using the explicit formula derived. In order to apply the technique of multiple coverage, the seismic traces are gathered in common conversion point (CCP) records. The coordinates of the CCP are calculated for a given depth and an *a priori* assumption on the average ratio v_p/v_s. In the near-offset range, the moveout correction can be calculated with sufficient accuracy by a hyperbolic approximation of the traveltime curve. Therefore, in this range, standard stacking software can be used to process the converted wave data and produce a stacked PS or SP time section.

The rms velocity has been defined for converted waves of simple PS- or SP-type and have been related to the product $v_p v_s$ of the interval velocities by a Dix-type formula for converted waves. In the application of this formula to obtain the ratio v_p/v_s, an essential requirement is the correct correlation of the events in the PP-section to those in the PS-section. This can introduce difficulties, especially when the conversion properties of the interfaces differ strongly from their reflection properties. Additional problems can be caused, for example by effects of anisotropy or by insufficient shear wave static corrections. Even though the determination of P- and S-wave interval velocities from PP- and PS-data may still encounter difficulties, we think that CCP stacking of PS-data can confirm or supplement structural information obtained from CMP stacking of PP-data.

Acknowledgements

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Appendix A
The Coordinates of the Conversion Point for a Single Horizontal Isovelocity Layer

Holling and Kües (1984) calculated the offset of a converted wave from the coordinates of the midpoint and the conversion point. In an analogous way we find the horizontal distance D of the conversion point from the midpoint.

From Fig. 9 we have

$$D = x_p - \frac{x}{2}, \tag{A1}$$

or, with $x_p = x - x_s$ and $x_s = z \tan \beta$,

$$D = \frac{x}{2} - z \tan \beta. \tag{A2}$$

From Snell's law and

$$\sin \beta = \frac{\tan \beta}{\sqrt{(1 + \tan^2 \beta)}}, \qquad 1 + \tan^2 \alpha = \frac{1}{\cos^2 \alpha},$$

one obtains

$$\tan \beta = \frac{\tan \alpha}{\sqrt{[(v_p/v_s)^2 + ((v_p/v_s)^2 - 1) \tan^2 \alpha]}}. \tag{A3}$$

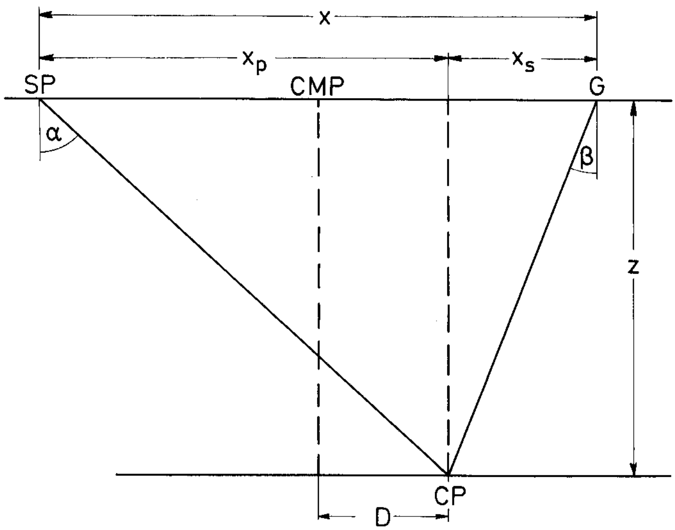

FIG. 9. The raypath of a PS-converted wave.

With

$$\tan \alpha = \frac{x_p}{z} = \frac{D + \dfrac{x}{2}}{z}, \tag{A4}$$

we have

$$\tan \beta = \frac{D + \dfrac{x}{2}}{z\sqrt{\left[(v_p/v_s)^2 + ((v_p/v_s)^2 - 1)\dfrac{(D + x/2)^2}{z^2}\right]}}. \tag{A5}$$

Substituting (A5) into (A2) gives

$$D = \frac{x}{2} - \frac{D + \dfrac{x}{2}}{\sqrt{\left[(v_p/v_s)^2 + ((v_p/v_s)^2 - 1)\dfrac{(D + x/2)^2}{z^2}\right]}}. \tag{A6}$$

Squaring (A6), we have

$$D^4 + \left(z^2 - \frac{x^2}{2}\right)D^2 - z^2 kxD + \tfrac{1}{16}(x^4 + 4x^2z^2) = 0, \tag{A7}$$

where $k = ((v_p/v_s)^2 + 1)/((v_p/v_s)^2 - 1)$. Numerical solutions of this equation have been presented by Behle and Dohr (1985).

With the substitutions

$$m = D/z \qquad \text{and} \qquad h = (x/2z)^2,$$

(A7) becomes

$$m^4 + (1 - 2h)m^2 - 2k\sqrt{h}\,m + (h + 1)h = 0. \tag{A8}$$

The roots of this equation are equal to the roots of

$$m^2 \pm \sqrt{[2(y + h) - 1]}\,m + y \pm \frac{2k\sqrt{h}}{2\sqrt{[2(y + h) - 1]}} = 0. \tag{A9}$$

Thus we obtain the solution

$$m_{1,2} = -\tfrac{1}{2}\sqrt{[2(y + h) - 1]} \pm \sqrt{\left[\tfrac{1}{2}(h - y - \tfrac{1}{2}) - \frac{k\sqrt{h}}{\sqrt{[2(y + h) - 1]}}\right]},$$

$$m_{3,4} = \tfrac{1}{2}\sqrt{[2(y + h) - 1]} \pm \sqrt{\left[\tfrac{1}{2}(h - y - \tfrac{1}{2}) + \frac{k\sqrt{h}}{\sqrt{[2(y + h) - 1]}}\right]}, \tag{A10}$$

where y is any of the roots of the cubic equation

$$y^3 + (h - \tfrac{1}{2})y^2 - (h^2 + h)y + \tfrac{1}{2}(-2h^3 - h^2 + (1 - k^2)h) = 0. \tag{A11}$$

Substituting $y = \tilde{y} - h/3 + \frac{1}{6}$ we get

$$\tilde{y}^3 + 3p\tilde{y} + 2q = 0 \tag{A12}$$

with

$$3p = -\tfrac{4}{3}h^2 - \tfrac{2}{3}h - \tfrac{1}{12}$$

and

$$2q = \tfrac{1}{27}(-16h^3 - 12h^2 - 3h + \tfrac{27}{2}(1 - k^2)h - \tfrac{1}{4}).$$

(A12) can be solved using Cardano's formula

$$\tilde{y} = \sqrt[3]{(-q + \sqrt{d})} + \sqrt[3]{(-q - \sqrt{d})}, \tag{A13}$$

with $d = q^2 + p^3$. (A11) has one real root and two complex roots because $d > 0$. The real root is used to find the roots given in (A10). The horizontal distance D of the conversion point from the midpoint between source point and receiver is

$$D = mz, \tag{A14}$$

where m can be found from (A10) with the condition

$$D \leq \frac{x}{2}$$

Thus the horizontal distance x_p of the conversion point from the source point is

$$x_p = \frac{x}{2} + D, \tag{A15}$$

for PS-converted waves. For converted SP-waves the conversion point is found from simple geometrical considerations:

$$x_s = \frac{x}{2} - D. \tag{A16}$$

Appendix B
Series Expansion for the Traveltimes of Converted Waves

Taner and Koehler (1969) have expanded the traveltime function of reflected PP- or SS-type waves in a horizontally layered medium into a power series

$$t_n^2(x) = c_1 + c_2 x^2 + c_3 x^4 + c_4 x^6 + \cdots. \tag{B1}$$

Here we want to give an analogous expansion for the traveltime function of converted waves.

The path of a wave which propagates in a horizontally layered medium and is converted at the nth interface is shown in Fig. 10. The offset x_{ps} is given by

$$x_{ps}(p) = \sum_{k=1}^{n} (x_{p_k} + x_{s_k}) = p \sum_{k=1}^{n} \left(\frac{h_k v_{pk}}{\sqrt{(1 - p^2 v_{pk}^2)}} + \frac{h_k v_{sk}}{\sqrt{(1 - p^2 v_{sk}^2)}} \right), \tag{B2}$$

where h_k is the thickness of the kth layer, v_{sk} is the shear wave velocity, and v_{pk} is the compressional wave velocity in the kth layer. p is the ray parameter and is given by

$$p = \frac{\sin \alpha_k}{v_{pk}} = \frac{x_{pk}}{s_{pk} v_{pk}} = \frac{\sin \beta_k}{v_{sk}} = \frac{x_{sk}}{s_{sk} v_{sk}}. \tag{B3}$$

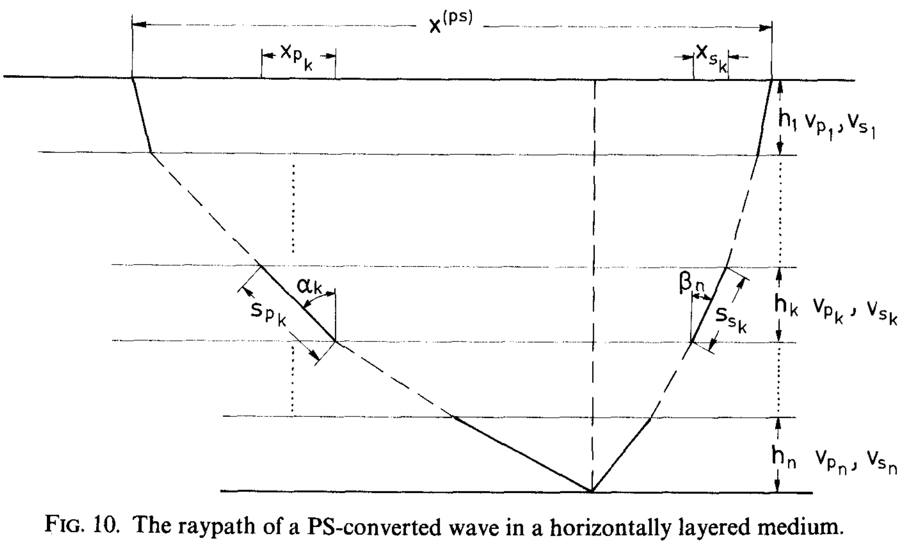

FIG. 10. The raypath of a PS-converted wave in a horizontally layered medium.

For the traveltime t_{ps} of the converted wave we get

$$t_{ps} = \sum_{k=1}^{n} \left(\frac{s_{pk}}{v_{pk}} + \frac{s_{sk}}{v_{sk}}\right), \tag{B4}$$

or

$$t_{ps}(p) = \sum_{k=1}^{n} \left(\frac{h_k/v_{pk}}{\sqrt{(1 - p^2 v_{pk}^2)}} + \frac{h_k/v_{sk}}{\sqrt{(1 - p^2 v_{sk}^2)}}\right). \tag{B5}$$

The function $(1 - p^2v^2)^{-1/2}$ can be expanded into a Taylor series as

$$(1 - p^2v^2)^{-1/2} = 1 + \frac{1}{2} p^2v^2 + \frac{1 \times 3}{2 \times 4} (p^2v^2)^2 + \frac{1 \times 3 \times 5}{2 \times 4 \times 6} (p^2v^2)^3 + \cdots. \tag{B6}$$

Substituting in (B2) and (B5) one obtains

$$x_{ps} = \sum_{j=1}^{\infty} q_j \sum_{k=1}^{n} h_k(v_{pk}^{2j-1} + v_{sk}^{2j-1})(p^{2j-1}), \tag{B7}$$

$$t_{ps} = \sum_{j=1}^{\infty} q_j \sum_{k=1}^{n} h_k(v_{pk}^{2j-3} + v_{sk}^{2j-3})(p^{2j-2}), \tag{B8}$$

with

$$q_1 = 1;\; q_k = \frac{1 \times 3 \times \cdots \times (2k-3)}{2 \times 4 \times \cdots \times (2k-2)}. \tag{B9}$$

Let

$$a_m^{(ps)} = \sum_{k=1}^{n} h_k(v_{pk}^{2m-3} + v_{sk}^{2m-3}), \tag{B10}$$

$$b_m = q_m a_{m+1}, \tag{B11}$$

and

$$\gamma_m = q_m a_m. \tag{B12}$$

Substituting (B9) to (B12) into (B7) and (B8) yields

$$x_{ps} = p \sum_{k=1}^{\infty} b_k p^{2k-2}, \tag{B13}$$

$$t_{ps} = \sum_{k=1}^{\infty} \gamma_k p^{2k-2}. \tag{B14}$$

The infinite series (B13) and (B14) are now substituted in (B1). The coefficients of the power series (B1) can be obtained by comparison with the coefficients of like powers of p^2. The equations for x and t for waves with symmetrical ray paths are of the same form as those for waves with asymmetrical ray paths.

Therefore we can use the formulae given by Taner and Koehler (1969) in their Appendix A to calculate the coefficients $c_i(a_m)$ recursively. The first coefficient c_1 is given by

$$c_1 = a_1^{(ps)2} = \left(\sum_{k=1}^{n} h_k\left(\frac{1}{v_{pk}} + \frac{1}{v_{sk}}\right)\right)^2 = t_{0_n}^{(ps)2}, \tag{B15}$$

which is the square of the vertical two-way traveltime of the converted wave. If the ratio of compressional and shear wave velocity is constant:

$$v_{pk}/v_{sk} = v_p/v_s = \text{constant}, \tag{B16}$$

(B10) can be written as

$$a_m^{(ps)} = \frac{1 + \left(\dfrac{v_s}{v_p}\right)^{2m-3}}{2} (a_m^{(pp)}), \tag{B17}$$

and we find

$$c_1^{(ps)} = \left(\frac{1 + \dfrac{v_p}{v_s}}{2}\right)^2 c_1^{(pp)}. \tag{B18}$$

For the second coefficient one obtains

$$c_2 = \frac{a_1^{(ps)}}{a_2^{(ps)}} = \frac{\sum_{k=1}^{n}\left(\dfrac{1}{v_{pk}} + \dfrac{1}{v_{sk}}\right)h_k}{\sum_{k=1}^{n}(v_{pk} + v_{sk})h_k} = \frac{1}{\tilde{v}_{ps_n}^2} \tag{B19}$$

where $\tilde{v}_{ps_n}$ is the rms velocity with respect to the nth reflector. For constant ratios of v_p/v_s and with (B17) we obtain

$$c_2^{(ps)} = \frac{v_p}{v_s} c_2^{(pp)}. \tag{B20}$$

Further coefficients can be derived by recursion.

The convergence properties of the traveltime expansion for converted waves are qualitatively the same as those for the non-converted reflections. This can be shown by substituting (B10) into proofs given by Al-Chalabi (1973).

Appendix C
A Dix-Formula for Converted Waves

The approximation of the PS-traveltime curve by a hyperbola led us to the definition of the rms velocity $\tilde{v}_{ps}$ for PS- or SP-converted reflections.

Squaring (5) we get

$$\tilde{v}_{ps_n}^2 = \frac{\sum_{k=1}^{n}(v_{pk} + v_{sk})h_k}{\sum_{k=1}^{n}\left(\dfrac{1}{v_{pk}} + \dfrac{1}{v_{sk}}\right)h_k}, \tag{C1}$$

and with the vertical two-way traveltime to the nth reflector

$$t_{0_n}^{(\mathrm{ps})} = \sum_{k=1}^{n} \left(\frac{1}{v_{\mathrm{p}k}} + \frac{1}{v_{\mathrm{s}k}}\right) h_k, \tag{C2}$$

we can write

$$\tilde{v}_{\mathrm{ps}_n}^2 = \frac{\sum_{k=1}^{n} (v_{\mathrm{p}k} + v_{\mathrm{s}k}) h_k}{t_{0_n}^{(\mathrm{ps})}}. \tag{C3}$$

The thickness h_k of the kth layer can be expressed in terms of the one-way vertical traveltimes in layer k, $\tau_{0_k}^{(\mathrm{p})}$ and $\tau_{0_k}^{(\mathrm{s})}$, of compressional and shear waves, respectively, and the respective interval velocities:

$$h_k = \tfrac{1}{2}(\tau_{0_k}^{(\mathrm{p})} v_{\mathrm{p}k} + \tau_{0_k}^{(\mathrm{s})} v_{\mathrm{s}k}). \tag{C4}$$

Furthermore we have

$$\frac{\tau_{0_n}^{(\mathrm{p})}}{\tau_{0_n}^{(\mathrm{s})}} = \frac{v_{\mathrm{s}n}}{v_{\mathrm{p}n}}. \tag{C5}$$

Substituting (C4) into (C3) and rearranging by use of (C5) gives

$$\tilde{v}_{\mathrm{ps}_n}^2 t_{0_n}^{(\mathrm{ps})} = \sum_{k=1}^{n} (\tau_{0_k}^{(\mathrm{p})} + \tau_{0_k}^{(\mathrm{s})}) v_{\mathrm{p}k} v_{\mathrm{s}k}. \tag{C6}$$

For the conversion at the $(n-1)$th interface we obtain accordingly

$$\tilde{v}_{\mathrm{ps}_{n-1}}^2 t_{0_{n-1}}^{(\mathrm{ps})} = \sum_{k=1}^{n-1} (\tau_{0_k}^{(\mathrm{p})} + \tau_{0_k}^{(\mathrm{s})}) v_{\mathrm{p}k} v_{\mathrm{s}k}. \tag{C7}$$

Subtraction of (C7) from (C6) yields

$$\tilde{v}_{\mathrm{ps}_n}^2 t_{0_n}^{(\mathrm{ps})} - \tilde{v}_{\mathrm{ps}(n-1)}^2 t_{0(n-1)}^{(\mathrm{ps})} = (\tau_{0_n}^{(\mathrm{p})} + \tau_{0_n}^{(\mathrm{s})}) v_{\mathrm{p}_n} v_{\mathrm{s}_n}, \tag{C8}$$

or

$$v_{\mathrm{p}_n} v_{\mathrm{s}_n} = \frac{\tilde{v}_{\mathrm{ps}_n}^2 t_{0_n}^{(\mathrm{ps})} - \tilde{v}_{\mathrm{ps}(n-1)}^2 t_{0(n-1)}^{(\mathrm{ps})}}{\tau_{0_n}^{(\mathrm{p})} + \tau_{0_n}^{(\mathrm{s})}}. \tag{C9}$$

With

$$\tau_{0_n}^{(\mathrm{p})} + \tau_{0_n}^{(\mathrm{s})} = t_{0_n}^{(\mathrm{ps})} - t_{0(n-1)}^{(\mathrm{ps})}, \tag{C10}$$

we get

$$v_{\mathrm{p}_n} v_{\mathrm{s}_n} = \frac{\tilde{v}_{\mathrm{ps}_n}^2 t_{0_n}^{(\mathrm{ps})} - \tilde{v}_{\mathrm{ps}(n-1)}^2 t_{0(n-1)}^{(\mathrm{ps})}}{t_{0_n}^{(\mathrm{ps})} - t_{0(n-1)}^{(\mathrm{ps})}}. \tag{C11}$$

(C11) is a Dix-Krey-type formula for converted waves from which the product of the interval velocities for P- and S-waves can be calculated recursively.

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