GEOPHYSICS, VOL. 54, NO. 1 (JANUARY 1989); P. 82–89, 4 FIGS., 3 TABLES.

Empirical relationships among seismic velocity, effective pressure, porosity, and clay content in sandstone

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ABSTRACT

We use a multivariate analysis to investigate the influence of effective pressure P_e, porosity ϕ, and clay content C on the compressional velocity V_p and shear velocity V_s of sandstones. Laboratory measurements on water-saturated samples of 64 different sandstones provide a large data set that was analyzed statistically. For each sample, relationships between effective pressure and V_p and V_s have been determined. All samples were well fit by relationships that have an exponential increase in velocity at low P_e, tapering to a linear increase with P_e for P_e greater than 0.2 kbar. There are differences in the pressure dependences of velocity for different rocks, particularly at very low pressures; however, the differences cannot be attributed to ϕ or C. For the combined set of measurements from all samples, the best fitting formulations are

$$V_p = 5.77 - 6.94\phi - 1.73\sqrt{C} + 0.446(P_e - e^{-16.7P_e})$$

and

$$V_s = 3.70 - 4.94\phi - 1.57\sqrt{C} + 0.361(P_e - e^{-16.7P_e}).$$

While this is admittedly a very simplified parameterization, it is remarkable how well the velocity of the rocks considered here can be predicted based on only the three parameters, ϕ, C, and P_e. The model accounts for 95 percent of the variance and has rms error of 0.1 km/s. An increased value of V_p/V_s can indicate a decrease in P_e, a decrease in porosity, or an increase in clay content or some combination thereof.

INTRODUCTION

In a laboratory environment, geophysical measurements can be made accurately for a rock sample of known characteristics. Since geophysicists commonly try to characterize rocks using measurements made in the field, we wish to investigate the combining of laboratory measurements to form empirical relationships that can be applied to estimating characteristics of in-situ rocks. For a given rock, seismic velocity has been shown to be a strong function of effective pressure P_e, that is, the confining pressure reduced by the pore pressure (Todd and Simmons, 1972). Since velocity also varies with the porosity and composition of the rock, an empirical relationship may permit us to estimate the effective pressure, and hence the pore pressure, from measurement of in-situ velocity. While the influence of porosity on velocity has been analyzed for many years with such formulas as the Wyllie time-average equation (Wyllie et al., 1956), Han et al (1986) sought to systematically investigate the additional effect of clay content in sandstones. They showed that even a small amount of clay significantly affects velocity, so that clay content must be considered when formulating an empirical velocity relationship.

Han (1986) reported compressional velocity V_p and shear velocity V_s for a broad suite of sandstones over a P_e range of 0.02 to 0.49 kbar. All samples were fully water-saturated; thus the influence of gas and oil mixtures on velocity was not included in these data. He measured velocities for 64 different samples, including clean quarry sandstones, other well-consolidated sandstones from quarries and boreholes, tight-gas sandstones, and samples (some poorly consolidated) from offshore wells in the Gulf of Mexico. The porosity ϕ ranged from 0.02 to 0.30 and was measured by a helium porosimeter.

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The clay content C ranged from 0.00 to 0.50 and was measured by point counting on two thin sections per sample, 300 counts per section. The velocities were measured with a pulse transmission technique with central frequencies of 1.0 MHz and 0.6 MHz for P and S waves, respectively. Absolute errors are estimated to be less than 1 percent in V_p and less than 2 percent in V_s. If we assume that these data provide a representative sample of sandstones in general, we can use them to find an empirical relationship describing velocity as a function of porosity, clay content, and effective pressure.

Han (1986) varied the confining pressure from 0.01 to 0.80 kbar, while the pore pressure varied from 0.00 to 0.40 kbar. Samples were prepressurized to 0.50 kbar to reduce hysteresis effects. For a specific sample at a given effective pressure, Han found very slight velocity differences for different values of confining pressure and pore pressure, suggesting that both V_p and V_s are essentially functions of effective pressure. Therefore, we combined the data from all confining pressures and pore pressures for each rock and considered velocity to be a function of the effective pressure only.

In this study, we first find a general relationship between velocity and P_e for each individual rock, ignoring the clay and porosity parameters. We show that the pressure dependences for the individual rocks are similar enough that we can estimate the pressure effect for the entire suite of rocks together. Our next step is to combine the entire set of velocity measurements for all rocks and find an empirical relationship based on the three parameters: clay, porosity, and effective pressure. Such a relationship is a step in the long-term process of understanding rock behavior; while various, sometimes competing, models of physical processes are being developed, an empirical relationship can provide a useful description, for a very large set of data, of the influence of effective pressure, porosity, and clay content on seismic velocity.

VELOCITY-PRESSURE RELATIONSHIPS FOR INDIVIDUAL ROCKS

For each rock sample with a given value of porosity and clay content, we use the relationship

$$V = A + KP_e - Be^{-DP_e}. \qquad (1)$$

This relationship can well describe the laboratory observations of velocity variation with effective pressure for pressures equivalent to those in the upper crust (0 to 1.5 kbar). Although used by others for crystalline rocks (Moos, 1983; Stierman et al., 1979), the relationship also works well for sedimentary rocks, since it describes well the velocity increase with effective pressure as microcracks and pores close and the rock becomes less compressible. As illustrated by the derivative of equation (1),

$$\frac{dV}{dP_e} = K + BDe^{-DP_e}, \qquad (2)$$

the velocity increases most rapidly as P_e is initially increased and a relatively large number of microcracks close. A larger value for B indicates the increased relative importance of crack closure, while a larger value for D indicates that the cracks close more rapidly as P_e is increased. Further increases in P_e are associated with more nearly linear increases in velocity.

To fit equation (1) to the data in this study, we apply a grid search over the range 1 to 40 for the exponential coefficient D, and for each D, calculate the least-squares solution for the other coefficients, A, K, and B. For each solution, a fit parameter is calculated based on the root-mean-square (rms) residual in V_p, V_s, shear modulus μ, and Poisson's ratio ν. The best-fitting set of coefficients is chosen to represent each rock sample. Most rocks had 17 measurements of both V_p and V_s; no solution was calculated for samples that had fewer than six measurements of both V_p and V_s. Table 1 lists the results for all the rocks studied. Since we were able to fit the observed velocities extremely well (Figure 1), this table substantially provides a summary of the nearly 2000 laboratory measurements.

Figure 1 shows the curves fit to V_p, V_s, and V_p/V_s data for six samples, representing a broad range of porosity, 0.059 to 0.261, and clay content, 0.00 to 0.45. For a given P_e, there are large variations in the velocities of these samples, resulting from the dependence on ϕ and C (as well as other unmodeled factors and experimental error). Comparing Figures 1e and 1f, we note that samples with similar ϕ and C have similar V_p values of 3.3 to 3.5 km/s at 0.2 kbar, whereas Utah Buff (Figure 1a), a low ϕ and low C sample, has a much higher V_p of 4.9 km/s at 0.2 kbar. As illustrated, we obtained for each sample, over the measured range of P_e, an excellent fit to the observed velocities using equation (1). The velocity increases linearly with P_e above approximately 0.2 kbar; at lower pressures, the rate of increase in velocity with pressure is much greater. The V_p/V_s ratio is largest at very low P_e and decreases as P_e increases. However, while the general pattern of behavior is common to all rocks measured, there are major variations among the individual samples. Compare Gulf124155, (e) which shows the largest pressure effect, with Utah Buff, (a) which shows the least. The shape of the curves is not simply dependent on ϕ or C. For example, the rocks in e and f have similar ϕ and C parameters, yet show distinctly different behavior at low P_e.

To investigate further whether the effects of pressure on velocity might be dependent on ϕ or C, we applied regression analysis to the coefficients in equation (1) and the parameters ϕ and C. None of the coefficients shows any statistically significant relationship to the parameters. Thus, the pressure dependence can be separated from the porosity and clay dependence and the most valid function for the exponential coefficient D is simply a constant, the average value. There is no significant difference between the values obtained for V_p and V_s coefficients. Thus, for both V_p and V_s, the same exponential coefficient can be used, $D = 16.7 \pm 5.3$ (for P_e in kbar).

REGRESSION ON ALL DATA

Our next step is to apply forward stepwise multiple regression on all the measured velocity data in order to obtain the best-fitting relationship of the form

$$V = f[\phi, C, P_e, e^{-DP_e}],$$

thus combining the observed dependence on effective pressure with Han et al.'s (1986) observation of clay content and poros-

Table 1. Velocity-pressure relationships for individual rocks.

	$V = A + KP_e - Be^{-DP_e}$									
			V_p				V_s			
Sample	ϕ	C	A	K	B	D	A	K	B	D
Beaver	0.064	0.00	5.47	0.199	0.503	9	3.44	0.381	0.399	11
Berea350	0.227	0.06	3.91	0.307	0.622	22	2.31	0.197	0.537	19
Berea400	0.222	0.03	3.82	0.346	0.441	18	2.32	0.208	0.353	12
Berea500	0.195	0.09	4.01	0.216	0.451	10	2.44	0.256	0.443	13
Boise	0.259	0.06	3.65	0.221	0.337	23	2.04	0.109	0.139	15
Coconino	0.111	0.06	4.68	0.146	0.187	9	2.90	0.243	0.141	13
Conotton	0.236	0.04	3.80	0.346	0.716	24	2.28	0.206	0.616	17
Delawarebr	0.111	0.05	4.53	0.500	0.397	16	2.69	0.474	0.407	18
Dellightgr	0.046	0.07	5.17	0.138	0.419	8	3.06	0.266	0.270	15
Deltanbuff	0.160	0.03	4.45	0.403	0.257	28	2.73	0.238	0.278	18
Fontainbl	0.156	0.00	4.70	0.294	0.532	35	3.00	0.232	0.518	30
Fontainblb	0.200	0.00	4.36	0.256	0.664	22	2.78	0.175	0.571	19
Gulf10379	0.144	0.44	3.58	0.372	0.492	26	1.85	0.291	0.365	27
Gulf10381	0.143	0.46	3.50	0.326	0.301	14	1.92	0.177	0.224	14
Gulf10392	0.132	0.51	3.46	0.511	0.425	19	1.93	0.194	0.399	16
Gulf10431	0.312	0.11	3.12	0.235	0.558	17	1.65	0.271	0.389	18
Gulf10432	0.305	0.12	3.11	0.197	0.435	14	1.73	0.107	0.333	13
Gulf10452V	0.111	0.41	3.86	0.313	0.429	16	2.14	0.127	0.390	13
Gulf12409B	0.158	0.29	3.72	0.720	0.608	22	1.96	0.488	0.625	21
Gulf12409	0.162	0.27	3.74	0.595	0.631	13	1.99	0.339	0.556	13
Gulf124155	0.256	0.22	3.20	0.396	0.732	19	1.80	0.252	0.643	13
Gulf12416	0.264	0.12	3.38	0.448	0.641	14	1.87	0.202	0.515	11
Gulf12418	0.155	0.37	3.64	0.326	0.502	15	2.00	0.295	0.418	12
Gulf124255	0.123	0.44	3.69	0.373	0.359	24	2.05	0.230	0.374	19
Gulf126605	0.159	0.27	3.69	0.367	0.466	19	2.00	0.353	0.437	20
Gulf12670	0.272	0.08	3.55	0.324	0.610	16	2.09	0.273	0.398	14
Gulf12674	0.276	0.06	3.45	0.414	0.713	18	1.98	0.291	0.774	17
Gulf12676	0.294	0.11	3.41	0.393	0.807	19	1.94	0.294	0.674	17
Gulf12677B	0.283	0.08	3.41	0.429	0.795	15	1.96	0.322	0.632	13
Gulf12677	0.275	0.07	3.32	0.475	0.716	13	1.81	0.440	0.804	21
Gulf14807	0.213	0.11	3.60	0.694	0.747	18	2.07	0.421	0.605	12
Gulf15845	0.127	0.21	4.00	0.637	0.682	17	2.38	0.317	0.409	6
Gulf15879	0.162	0.06	4.37	0.608	0.776	12	2.41	0.757	0.681	18
Gulf158XX	0.117	0.23	4.17	0.625	0.568	14	2.37	0.588	0.485	12
Gulf15930	0.069	0.24	4.42	0.446	0.675	14	2.57	0.424	1.270	28
Gulf15949	0.161	0.18	3.89	0.464	0.452	14	2.22	0.372	0.451	16
Indianada1	0.266	0.16	3.20	0.353	0.308	12	1.92	0.159	0.305	10
Indianada2	0.261	0.16	3.37	0.460	0.323	20	1.91	0.367	0.301	25
Indianali1	0.240	0.10	3.70	0.049	0.366	7	2.12	0.258	0.294	16
Indianali2	0.245	0.10	3.48	0.550	0.380	21	2.04	0.319	0.390	20
Massburgun	0.243	0.03	3.71	0.501	0.507	24	2.26	0.277	0.320	19
Massildar1	0.184	0.06	4.21	0.325	0.378	15	2.54	0.216	0.369	13
Massildar2	0.184	0.06	4.16	0.357	0.343	16	2.47	0.244	0.345	17
Massillon	0.212	0.04	3.94	0.241	0.646	12	2.25	0.365	0.522	17
Nuggeth	0.097	0.08	4.76	0.315	0.239	13	2.93	0.261	0.308	15
Nuggetv	0.096	0.08	4.59	0.277	0.364	13	2.77	0.418	0.322	20
P615561	0.073	0.38	4.30	0.207	0.375	15	2.55	0.192	0.386	21
P636249	0.080	0.40	4.17	0.239	0.271	19	2.38	0.272	0.377	31
P636254	0.121	0.37	3.95	0.382	0.464	24	2.23	0.294	0.424	27
P646256	0.098	0.40	4.21	0.141	0.323	8	2.44	0.204	0.296	13
P646258	0.103	0.35	4.06	0.331	0.299	14	2.31	0.313	0.330	21
P646260	0.077	0.45	4.22	0.304	0.387	13	2.45	0.275	0.376	15
P646264	0.147	0.13	4.44	0.241	0.742	7	2.55	0.259	0.575	11
P727154	0.170	0.14	4.13	0.499	0.380	21	2.37	0.436	0.548	30
P748797	0.162	0.10	4.03	0.586	0.472	15	2.36	0.377	0.464	12
P827377	0.180	0.11	4.07	0.409	0.572	13	2.31	0.317	0.509	11
P827379	0.177	0.16	4.00	0.486	0.478	13	2.26	0.416	0.515	17
Redstone	0.167	0.28	3.66	0.426	0.338	19	2.01	0.173	0.357	19
Stabarbara	0.131	0.27	3.92	0.364	0.539	14	2.11	0.327	0.484	17
StPeter1	0.205	0.00	4.21	0.187	0.746	24	2.58	0.160	0.781	20
StPeter2	0.187	0.00	4.55	0.298	0.800	19	2.83	0.195	0.741	16
Torrey	0.136	0.14	4.17	0.138	0.467	6	2.22	0.439	0.255	13
Unionh	0.194	0.05	4.03	0.411	0.669	19	2.39	0.280	0.535	15
Utahbuff	0.059	0.06	4.86	0.201	0.109	14	3.05	0.193	0.123	17

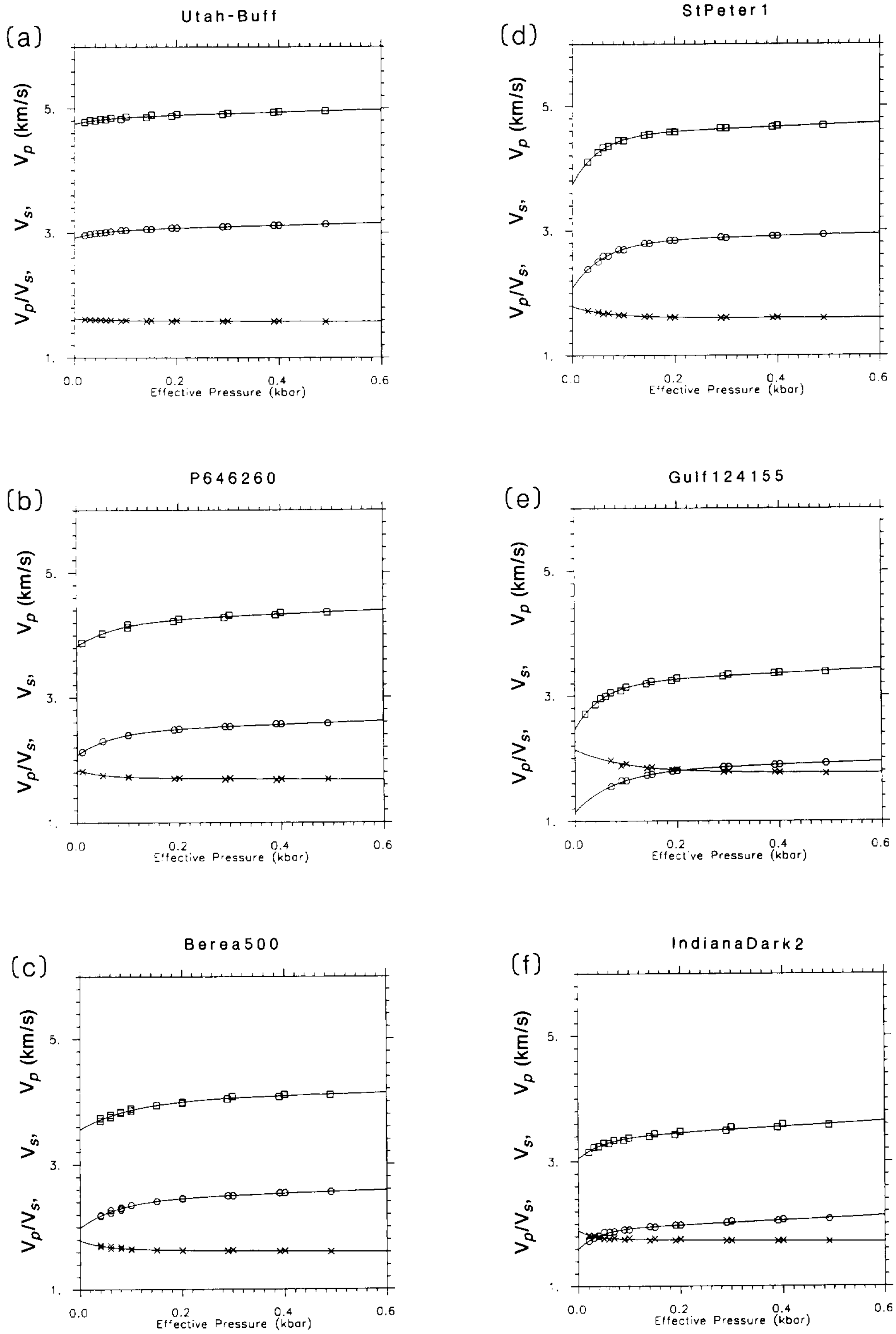

FIG. 1. Plots of calculated velocity-pressure relationships for six representative samples. Circles are measured velocities and stars are measured V_p/V_s ratios. (a) Utah Buff, $\phi = 0.059$, $C = 0.06$; (b) P646260, $\phi = 0.077$, $C = 0.45$; (c) Berea500, $\phi = 0.195$, $C = 0.09$; (d) StPeter1, $\phi = 0.205$, $C = 0.00$; (e) Gulf124155, $\phi = 0.256$, $C = 0.22$; (f) IndianaDark2, $\phi = 0.261$, $C = 0.16$.

ity dependence. All the velocity data are used together instead of being separated by rock sample as was done in the preceding section, or by pressure as was done by Han et al. (1986). After searching over possible relationships that included various functions of each parameter, as well as combinations of parameters, the best fit was found for

$$V = B_0 + B_1\phi + B_2\sqrt{C} + B_3 P_t, \qquad (3)$$

where the effective pressure is transformed to

$$P_t = P_e - e^{-DP_e} \qquad (4)$$

with the exponential coefficient D determined in the preceding section. We initially used a transformation $P_t = P_e - \beta e^{-DP_e}$, similar to equation (1), but β turned out to be unity to within the rms error of the coefficient. Additional terms in equation (3) did not significantly improve the fit to the data. The square root helps account for the effect on velocity of the initial clay addition; thus, adding 4 percent clay to clean sandstone has the same effect as adding 20 percent clay to a rock that originally had 10 percent clay. Han et al. (1986), in regression for ϕ and C on a smaller number of data points at one pressure, tried to adjust for this effect by separating out the samples with no clay. In our study, using all data from all pressures together, the $\sqrt{C}$ parameter was still significantly better than C even when the clean sandstones were removed. We also tried other clay exponents from $C^{0.25}$ to $C^{0.90}$; however, $\sqrt{C}$ turned out to be the best parameter for V_p and V_s.

We find that V_p and V_s can be described by

$$V_p = 5.77 - 6.94\phi - 1.73\sqrt{C} + 0.446(P_e - e^{-16.7P_e}) \qquad (5)$$

and

$$V_s = 3.70 - 4.94\phi - 1.57\sqrt{C} + 0.361(P_e - e^{-16.7P_e}), \qquad (6)$$

Table 2. Linear regression solution (P_e in kbar).

$$V = B_0 + B_1\phi + B_2\sqrt{C} + B_3(P_e - e^{-16.7P_e})$$

	Parameter	Coefficient	rms error ofcoeff.	F-value*
		P velocity		
B_0		5.771		
B_1	porosity	−6.938	0.052	17 937
B_2	claysqrt	−1.725	0.020	7 238
B_3	$P_e - e^{-DP_e}$	0.446	0.010	2 034
		S velocity		
B_0		3.704		
B_1	porosity	−4.937	0.050	9 649
B_2	claysqrt	−1.568	0.019	6 635
B_3	$P_e - e^{-DP_e}$	0.361	0.010	1 324

*Note: critical F-value is 4.00.

Table 3. Fit of regression model.

	rms error (km/s)	Reduction in data variance
V_p	0.105	96%
V_s	0.099	94%

with P_e in kbar. The coefficients, along with their rms errors and F-test values, are listed in Table 2. The size of the coefficient shows how strongly a given parameter influences velocity, since ϕ, C, and P_t have roughly the same numerical range, whereas the F-value shows how statistically significant that parameter is in the regression. The porosity and clay terms have the largest effects, but the effective pressure term is also highly significant, as shown by the F-values. The total reduction in data variance is 96 percent for V_p and 94 percent for V_s (Table 3). The predicted velocities and residuals are shown for V_p in Figures 2a and 2b and for V_s in Figures 2c and 2d. It is remarkable, considering that rocks are complex systems, how well velocity of the rocks considered here can be predicted based on only the three parameters ϕ, C, and P_e. In Figures 2a and 2c, the points cluster tightly around a 45° line. There are no extreme outliers and the largest residual is only 0.35 km/s, about 12 percent of the range of measured variation. The rms errors for both V_p and V_s are about 0.1 km/s; thus, equations (5) and (6) fit the measured velocities reasonably well. Figure 2b and 2d show the residuals plotted at a scale five times that of Figures 2a and 2c so that different types of residual patterns can easily be observed, and lines are drawn that enclose plus and minus two rms errors.

DISCUSSION

There are a few distinct series of points in Figure 2 that systematically do not fit the line and correspond to particular rocks. We can consider how well equations (5) and (6) predict the velocities for an individual rock sample by looking at the residuals for the data that are fit most poorly. In the residual plot, the average velocity residual for a particular sample is related to how well the empirical coefficients for porosity and clay describe that rock's behavior, since porosity and clay are constant for a given rock sample. Patterns in the residuals for an individual sample reflect discrepancies between the velocity variation with P_e of the empirical model and that of the particular rock sample. In Figure 2b, samples are highlighted that have numerous large V_p residuals and/or unusual trends in residuals. For Utah Buff, which has very little variation with pressure (Figure 1a), the average predicted velocity is about the same as the average observed velocity; however, the predicted velocity is too low at low pressures and too high at high pressures, thus creating a distinct pattern. In contrast, for Gulf12677 all predicted velocities are too high: the residual for one measurement is greater than three rms errors. There is also a strong pattern in the residuals, since this sample has larger variation with pressure in both the linear K and exponential B terms than the regression relationship expects. Despite having very similar characteristics to Gulf12677, the sample Gulf12676 has predicted velocities that are all too low. The trend of residuals is also less pronounced with fairly uniform residuals for P_e above 0.06 kbar, since the linear part of the velocity variation with P_e is slightly less than that of the regression model, while the exponential decay is larger.

StPeter1 shows a combination of the residual trends observed for other rocks. It has a smaller linear increase with P_e, yet a larger exponential decrease at low P_e than most of the samples (Figure 1d). P727154 and Berea500 do not exhibit particular trends in their residuals, since the empirical V_p-P_e relationship fits these samples well. P727154 has uniformly

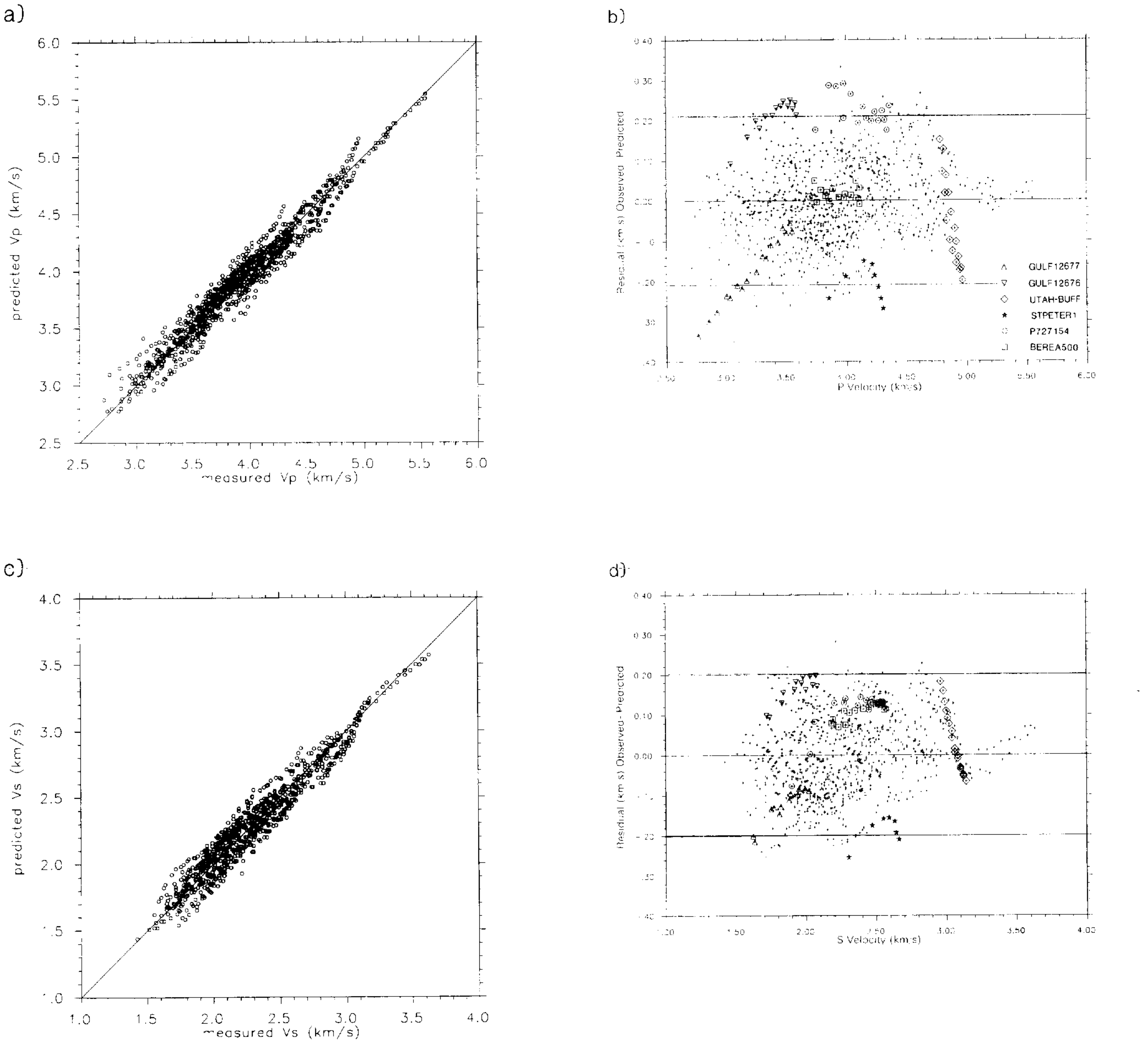

FIG. 2. Velocity predictions using equations (5) and (6). (a) Predicted versus measured V_p, (b) V_p residuals (observed − predicted) versus measured V_p, with samples marked that have numerous large residuals or unusual trends in residuals. Lines indicate ± 2 rms error, (c) Predicted versus measured V_s, (d) V_s residuals versus measured V_s, with the same samples marked as in (b).

large positive residuals, while Berea500 is one of the best-fitting rocks with uniformly small residuals. Thus the porosity and clay terms are underestimating velocity for P727154, but are providing fairly accurate estimates for Berea500. Figure 2d shows the V_s residuals with the same rocks highlighted. The average residuals and residual patterns are similar to those for V_p. A difference is that Berea500 has larger shear velocities than predicted. P727154 shows a much larger exponential decrease in V_s at P_e less than 0.05 kbar, but this may be partly due to greater difficulty in measuring shear velocity at low effective pressures (Han, 1986).

All these effects are results of factors, such as grain and pore size and shape and degree of compaction and cementation, which are not included in our simple empirical relationship. Microstructural variations can particularly affect velocity at low P_e, even for rock samples with exactly the same composition and porosity (Bourbie and Zinszner, 1985). Additionally, the simple measurement of percent clay does not fully describe the rock's composition. However, the very good fit we obtain (Table 3) shows that the empirical relationship accounts for most of the velocity variation within our sample observations, and thus may be a useful tool for estimating velocity within the uncertainty indicated in Figure 2 for similar rocks that have no available laboratory measurements. It would be difficult to adjust for additional complexity in any simple uniform manner, as is illustrated by Gulf12676 and Gulf12677, which are very similar samples but have distinctly different patterns of residuals.

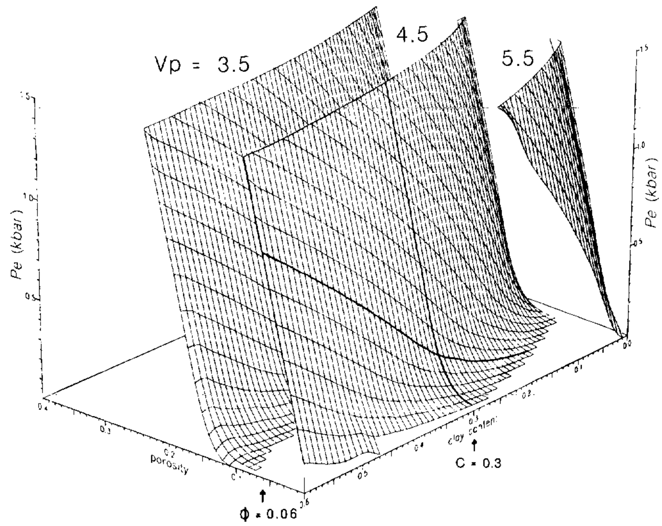

FIG. 3. Level surfaces, in the ϕ, C, P_e coordinate system, for increasing V_p. $V_p = 3.50$, $V_p = 4.50$, and $V_p = 5.50$. Most measurements of velocity at increasing pressures show that at some point between 1 and 2 kbar, the linear increase with pressure becomes less pronounced and above 5 kbar, the curves are nearly flat (Christensen, 1984). Thus, effective pressure is extrapolated to 1.5 kbar.

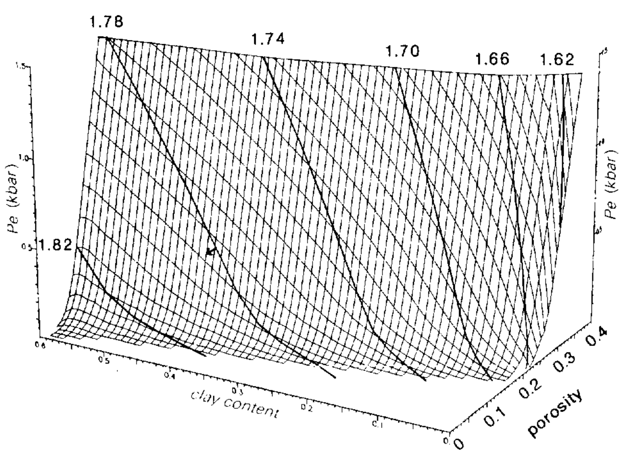

FIG. 4. Plot showing effect of varying V_p/V_s ratio for $V_p = 4.00$. Lines represent intersections with various V_s surfaces. Number next to line indicates the corresponding V_p/V_s ratio.

In order to visualize a function of three variables, in Figure 3 we show level surfaces of $V_p(\phi, C, P_e) = 3.5$ km/s, 4.5 km/s, and 5.5 km/s in the ϕ, C, P_e coordinate system. Thus a rock with a V_p of 4.5 km/s should be characterized by some point on the $V_p = 4.5$ surface. The surfaces are curved because of the $\sqrt{C}$ term and flatten at low effective pressure because of the exponential pressure term. The surfaces for different velocities are quite distinct. In these plots, the grid lines represent values of constant porosity and clay content. For example, the bold lines on the $V_p = 4.5$ surface are lines for $\phi = 0.06$ and $C = 0.30$; a rock with $\phi = 0.06$, $C = 0.30$, and $V_p = 4.5$ would be a point on the surface at the intersection of these lines, with estimated P_e of 0.23 kbar. For a relatively high V_p of 5.5 km/s, fairly large effective pressures are indicated except for extremely pure, low-porosity sandstones.

In Figure 4, we observe the influence of the V_p/V_s ratio by plotting intersection lines of various V_s level surfaces on the V_p level surface. The combinations of (ϕ, C, P_e) that predict a given V_p and V_s using our relationships are described by the intersection of the two surfaces. For pressures above the exponential decay, the V_p/V_s ratio seems to be most correlated with the porosity, since the V_p/V_s lines are generally aligned with lines of constant porosity, except for low clay content, where small amounts of clay greatly influence the elastic moduli. A normal to a V_p/V_s line (arrow in Figure 4) shows that an increase in the V_p/V_s ratio indicates a decrease in effective pressure, as is commonly observed, and/or an increase in the clay content or a decrease in porosity.

It is difficult to graph the rms error of the regression model, but it could be visualized as an envelope around a surface of Figure 3. Thus within the rms error there is, rather than a single point, a range of values that could indicate a certain velocity. For example, $V_p = 4.00$ km/s could be indicated by $\phi = 0.20 \pm 0.02$, $C = 0.25 \pm 0.05$, and $P_e = 0.62 \pm 0.23$.

CONCLUSIONS

For each of 64 different water-saturated sandstones, relationships between effective pressure and V_p and V_s have been determined. All samples could be well fit by relationships that have an exponential increase in velocity at low P_e, tapering to a linear increase with P_e at higher P_e. Variations in the pressure dependence among the samples do not correlate with porosity or clay content. The most useful form, considering all the samples, is ΔV proportional to $(P_e - e^{-16.7P_e})$, where P_e is in kbar.

We have applied forward stepwise multiple regression on the entire combined data set to find the best-fitting empirical relationships. These are

$$V_p = 5.77 - 6.94\phi - 1.73\sqrt{C} + 0.446(P_e - e^{-16.7P_e})$$

and

$$V_s = 3.70 - 4.94\phi - 1.57\sqrt{C} + 0.361(P_e - e^{-16.7P_e}).$$

The reduction in data variance is about 95 percent and the rms error is about 0.1 km/s. Since such complicating factors as grain size and pore shape distributions, amount of compaction, and degree of cementation are not described in the simple characterization by porosity and clay content, some samples are poorly fit by these empirical equations. Residual plots show that poorly fit rocks have uniformly large residuals due to a poor fit in the ϕ and C terms or unusual trends in residuals due to greater or lesser P_e variation than the typical sample.

The empirical relationships are able to provide a useful description, for a very large set of data, of the influence of effective pressure, porosity, and clay content on seismic velocity. While the relationships cannot exactly describe the velocity for all sandstones, the fit is reasonable for these varied samples, and thus may be useful for estimating the velocity of rocks for which laboratory measurements are unavailable.

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