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Short Note

The point of *P-S* mode-converted reflection: An exact determination

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INTRODUCTION

The accurate placement of a *P-S* mode-converted reflection point on a horizontal reflector is important in stacking surface seismic data for enhancement (Dohr and Janle, 1980; Garotta et al., 1985) and in interpreting results from Zoeppritz's or Knott's equations (Tooley et al., 1965; Young and Braile, 1976). This note describes an adaptation of Snell's law to determine exactly the reflection point of a *P-S* mode-converted reflection.

Figure 1 illustrates the definitions of geometric terms which will be used in the equations. The only nongeometric term is

$$G = V_s/V_p \qquad \{0 < G < 1\}, \tag{1}$$

where V_s and V_p are the velocities for an S wave and a P wave, respectively, in the layer above the reflector.

METHOD

Equation (2), Snell's law for a *P-S* mode-converted reflection, is

$$\sin(r_s) = G \sin(i_p). \tag{2}$$

Restating Snell's law in terms of geometric variables gives

$$\frac{X}{[X^2 + (Z_r - Z_g)^2]^{1/2}} = G \frac{(X_s - X)}{[(X_s - X)^2 + Z_r^2]^{1/2}}. \tag{3}$$

Equation (3) is easily rearranged into the quartic power series

$$X^4 + aX^3 + bX^2 + cX + d = 0, \tag{4}$$

where

$$a = \frac{2X_s(G^2 - 1)}{(1 - G^2)},$$

$$b = \frac{Z_r^2 - G^2(Z_r - Z_g)^2 - G^2X_s^2 + X_s^2}{(1 - G^2)},$$

$$c = \frac{2G^2X_s(Z_r - Z_g)^2}{(1 - G^2)},$$

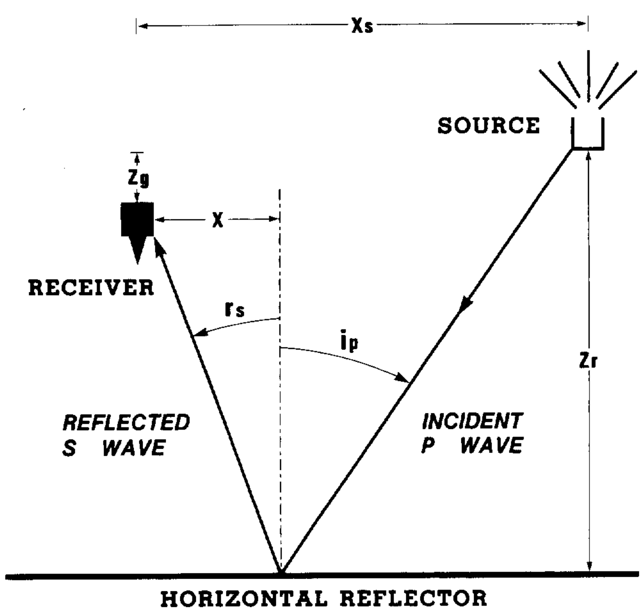

FIG. 1. Definitions of geometric terms used in equations (4) through (A-9). Positive numbers are measured radially away from the receiver on the horizontal axis and downward on the vertical axis. The layer above the horizontal reflector is assumed to be homogeneous and isotropic for both S waves and P waves.

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and

$$d = \frac{-G^2 X_s^2 (Z_r - Z_g)^2}{(1 - G^2)}.$$

Two of the four possible solutions of equation (4) are real numbers and two are complex numbers. The solution which corresponds to the *P-S* mode-converted reflection point is a real number corresponding to a point on a line between source and receiver; i.e.,

$$0 \leqslant X \leqslant X_s. \tag{5}$$

If numerical models show the same two solutions to be consistently complex and a third to be outside of boundary condition (5), the remaining solution uniquely determines the reflection point. Appendix A, after Speigal (1968), contains equations for each of the four possible solutions X_1 to X_4 and details on their derivations.

RESULTS

The first results from a series of numerical models using equation (4) showed that unless the depth variable Z_r is unity (1 km), the power series coefficients may become undefined. This must be dealt with by conditioning the input variables such that Z_r is unity and rescaling the output.

Figure 2 illustrates the results of a model simulating a VSP configuration in which the depth of the receiver is varied through the full range of stability. The bounds of applicability to the VSP problem are diagonally cross-hatched. The X_1

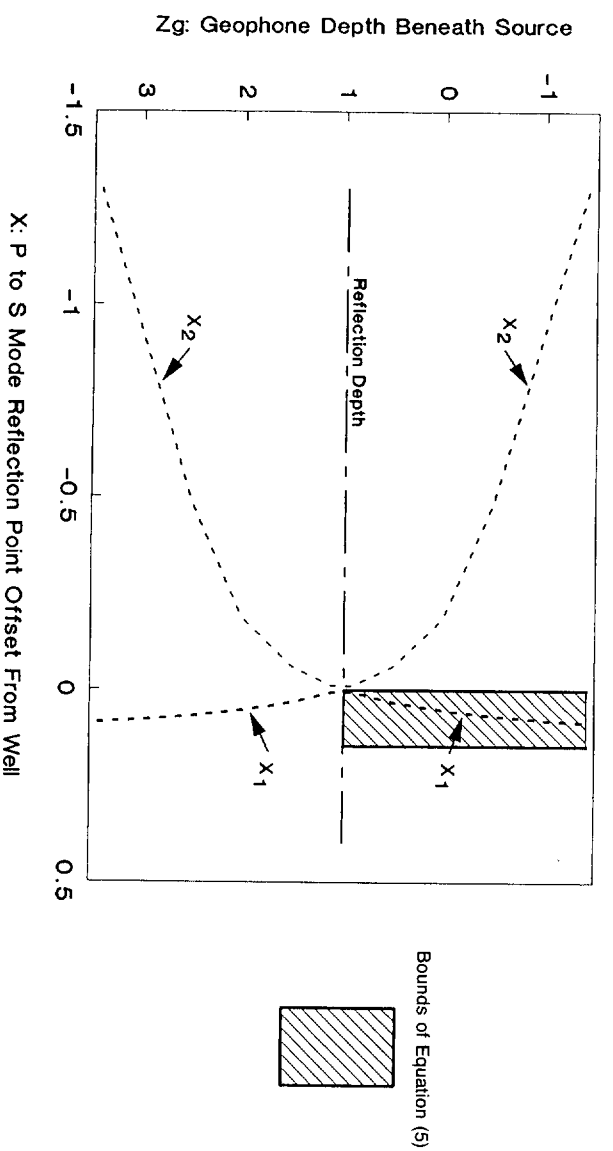

FIG. 2. Real solutions X_1 and X_2 to equation (4) for a VSP model. Source-to-receiver offset X_s = 0.15 km; horizontal reflector depth Z_r = 1.0 km; ratio (G) V_s/V_p = 0.57.

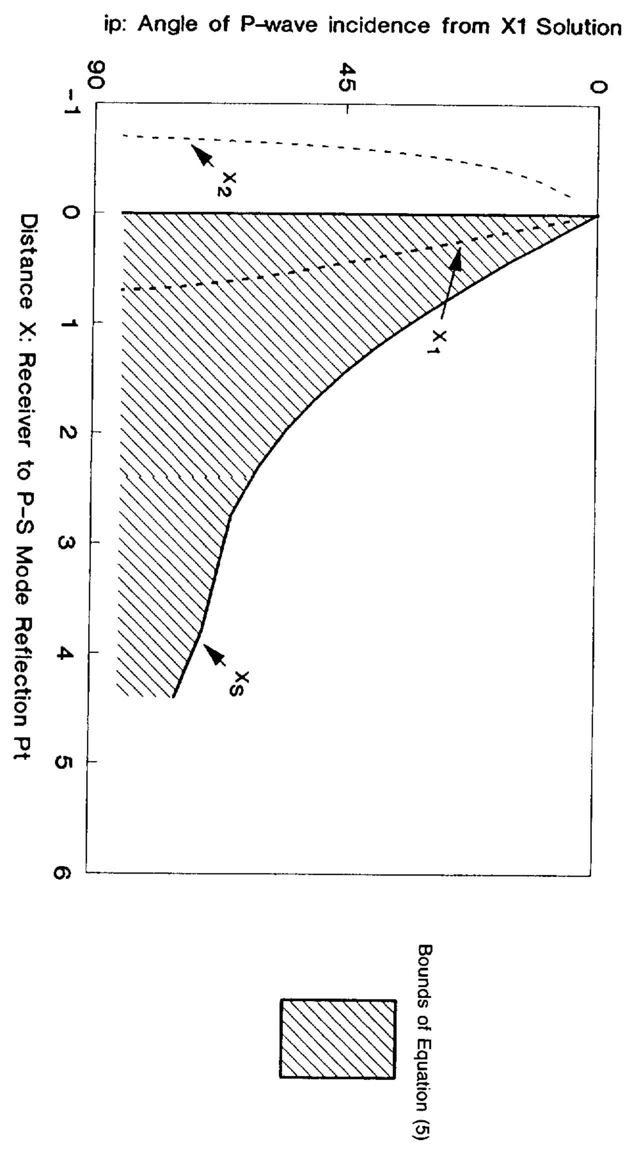

FIG. 3. Real solutions X_1 and X_2 to equation (4) for a surface seismic model; angles are in degrees. Geophone depth beneath source Z_g = 0; horizontal reflector depth Z_r = 1.0 km; ratio (G) V_s/V_p = 0.57.

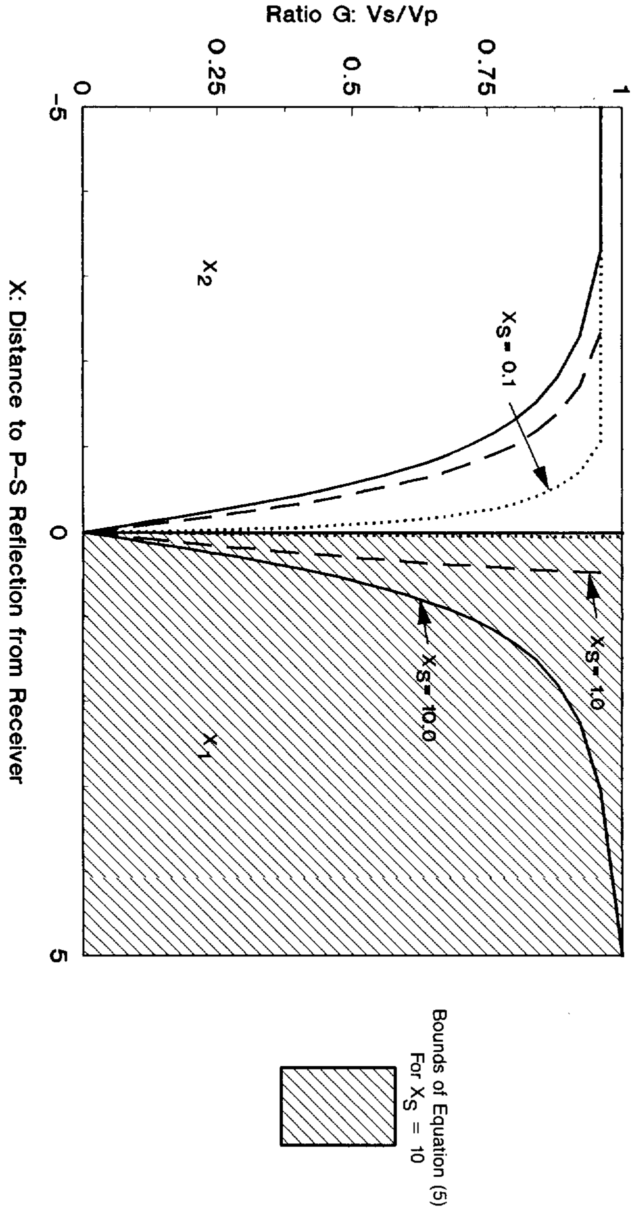

FIG. 4. Real solutions X_1 and X_2 for a range of S-wave to P-wave velocity ratios. Source-to-receiver offset X_s = {10, 1.0, 0.1} in km; receiver depth Z_g = 0; reflector depth Z_r = 1.0 km.

solution is the only solution conforming to condition (5). The X_2 solution is negative; X_3 and X_4 are complex conjugates.

Figure 3 illustrates the results of a model anticipating a range of possible surface seismic recording configurations. Figure 3 also shows that only solution X_1 produces a number which conforms to boundary condition (5). The other real number, X_2, is negative. Solutions X_3 and X_4 are complex conjugates.

Figure 4 illustrates the results of a surface seismic model anticipating the range of possible V_s/V_p ratios. For all such ratios, the only real solution conforming to condition (5) is the X_1 solution.

CONCLUSION

A wide range of appropriate parameters for both VSP and surface seismic recording configurations is modeled. These models demonstrate that the solution for Snell's law in the form of equation (4) which corresponds to X_1, equation (A-6) of Appendix A, is the only solution which conforms to boundary condition (5) and exactly determines the point of P-S mode-converted reflection from a horizontal reflector.

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APPENDIX A

SOLUTIONS TO EQUATION (4)

Solutions to equation (4) may be determined by finding any real solution to equation (A-1)

$$Y^3 + pY^2 + qY + r = 0, \tag{A-1}$$

where

$$p = -b,$$

$$q = ac - 4d,$$

and

$$r = 4bd - c^2 - a^2d.$$

The possible solutions to equation (A-1) are described by equations (A-2) through (A-4). In the range of conceivable VSP or surface seismic models where Z_r = 1.0, equation (A-2) may be relied upon to be a real number:

$$Y_1 = S + T - (p/3), \tag{A-2}$$

$$Y_2 = -\{[(S + T)/2] - [p/3]\} + j/2\{3^{1/2}[S - T]\}, \tag{A-3}$$

and

$$Y_3 = -\{[(S+T)/2] - [p/3]\} - j/2\{3^{1/2}[S-T]\}, \quad \text{(A-4)}$$

where

$$Q = [(3q - p^2)]/9,$$

$$R = [(9pq) - (27r) - (2p^3)]/54,$$

$$S = [R + (Q^3 + R^2)^{1/2}]^{(1/3)},$$

$$T = [R - (Q^3 + R^2)^{1/2}]^{(1/3)},$$

and

$$j = (-1)^{1/2}.$$

The four solutions of equation (4) are found in the two quadratic equations

$$X^2 + mX + n = 0$$

and (A-5)

$$X^2 + uX + v = 0,$$

where

$$m = [a + (a^2 - 4b + 4Y_1)^{1/2}]/2,$$

$$n = \{Y_1 - [(Y_1)^2 - (4d)]^{1/2}\}/2,$$

$$u = [a - (a^2 - 4b + 4Y_1)^{1/2}]/2,$$

and

$$v = \{Y_1 + [(Y_1)^2 - (4d)]^{1/2}\}/2.$$

Particular solutions to equations (A-5) and (4) are

$$X_1 = [-m + (m^2 - 4n)^{1/2}]/2, \quad \text{(A-6)}$$

$$X_2 = [-m - (m^2 - 4n)^{1/2}]/2, \quad \text{(A-7)}$$

$$X_3 = [-u + (u^2 - 4v)^{1/2}]/2, \quad \text{(A-8)}$$

and

$$X_4 = [-u - (u^2 - 4v)^{1/2}]/2. \quad \text{(A-9)}$$